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THE ANALYSIS OF ELASTIC-PLASTIC DEFORMATION AND STRESS
AT FINITE STRAIN AND THEIR EVALUATION

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Several aspects of elastic-plastic analysis relevant to technological applications which have been investigated on the project are discussed. References to the reports or papers dealing with these aspects are given. The topics covered are: (i) the generation of residual stress in metal-forming processes, in particular extrusion; (ii) finite-deformation elastic-plastic theory based on the nonlinear coupled kinematics; (iii) stress analysis in the presence of anisotropic hardening, in particular kinematic hardening and (iv) computer program development to improve accuracy and generality.

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## THE ANALYSIS OF ELASTIC-PLASTIC DEFORMATION AND STRESS AT FINITE STRAIN AND THEIR EVALUATION

### 1. THE GENERATION OF RESIDUAL STRESSES IN METAL-FORMING PROCESSES

A research program has been in progress for some years at Stanford University on the evaluation of stress and deformation distributions produced in metal-forming processes. The objective is to be able to compute the history of stress and deformation in the workpiece for different forming tool designs, billet material properties and process characteristics so that limits on these variables can be predicted which will ensure a sound formed product. This involves the avoidance of metal-forming defects such as the initiation and growth of internal or surface cracks or the generation of localized regions of high or uneven strain. The complete stress distributions include the residual stresses generated in the product by the forming process so that an objective of the program is also to determine these and elucidate how their magnitude and distribution is influenced by the process variables.

In cold working the yield stress of the workpiece is higher than for warm or hot working so that the residual stresses produced in cold working are in general higher. The study of the generation of residual stresses carried out on this project was therefore limited to cold-working processes. For most materials used for mechanical components or structural elements this means that rate-independent plasticity theory is appropriate in contrast to warm forming for which rate-dependent plasticity laws usually govern the process. Classical type plasticity theory with a yield condition and strain hardening will therefore be utilized. However, since large deformations and

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rotations occur in metal-forming processes, the formulation adopted must be valid for finite strains and rotations.

It is of course well known that residual stresses generated by the method of production of a part, as well, of course, as the mechanical characteristics generated, can play an important role in the utilization of the part and its reliability in service. If a part is to be machined, removal of stressed material will lead to residual deformation of the component which will increase with increasing stress magnitudes. Residual stresses can have a deleterious or beneficial effect on fatigue strength particularly in a corrosive atmosphere. Hence, investigation of the generation of residual stresses in metal-forming can be important from the standpoint of either avoiding defects by reducing residual stresses or tailoring the die-geometry to produce high beneficial stresses.

Reference [1] is a report written on this project which examines the generation of residual stress in extrusion. It was presented at the 28th Sagamore Army Materials Research Conference in July 1981 and will appear in the Proceedings. The paper illustrates the influence of die geometry on the residual stress distribution generated.

#### 2. FINITE-DEFORMATION ELASTIC-PLASTIC THEORY

Structural metals can often be deformed to large strains without fracturing. The rational design of many engineering processes and structures demands stress and deformation analysis in the presence of finite strain. In order to anticipate and hence prevent the generation of cracks, high residual stresses or other forming defects in the manufacture

of structural components, stress analysis must apply throughout a body and so elastic-plastic theory must be utilized. The neglect of the small elastic strain compared with plastic strain, and hence the adoption of rigid-plastic theory, prevents the determination of stresses in the rigid regions which may well comprise most of the work-piece in a metal forming process for example.

The early development of elastic-plastic theory was based on infinitesimal deformation theory on the basis of which the total strain was equal to the sum of elastic and plastic components with a similar summation law applying also for strain rates. At finite strains there is a coupling between elastic and plastic deformation since plastic flow occurs in a material already stressed to yield, and hence subject to elastic strains, and these two components interact in the nonlinear kinematics of finite-deformation theory. In the currently commonly used approach to finite-deformation analysis, the summation of elastic and plastic strain rates to give the total strain rate is adopted. The significance of this assumption is examined in the light of the nonlinear kinematical theory. The latter gives a precision to the kinematics which permits many aspects of the theory to be investigated more succinctly.

These developments were discussed in [2], a paper issued on the continuing project at R.P.I. and presented as a keynote address at the Eleventh Southeastern Conference on Theoretical and Applied Mechanics (SECTAM XI). It was pointed out that the incremental elastic-plastic theory developed in [3] on the preceding project could be interpreted as a purely deductive determination of the elastic-plastic operator relation appropriate when the Jaumenn derivative of stress is used for the stress

rate. The complete elasticity law, valid for finite deformation, is differentiated to provide the elastic strain-rate term to be substituted into the nonlinear kinematic relation along with the plastic strain-rate term to generate the stress rate-strain rate elastic-plastic operator relation. In contrast the usual approach based on Hooke's law is simply to express this in rate form by introducing the material velocity field, v(x,t), and then choosing an objective stress-rate expression which must be selected from an infinite number of possibilities. By utilizing the deductive approach, an operator stress rate-strain rate relation appropriate for a feasible stress rate definition is generated and its dependence on the stress-rate definition selected will be evident.

#### 3. ANISOTROPIC HARDENING

In an intriguing paper [4], Nagtegaal and de Jong evaluated the stresses generated by simple shear to large deformation in elastic-plastic and rigid-plastic materials which exhibit anisotropic hardening. In conformity with current practice for finite deformation in the case of kinematic hardening, they used an evolution equation for the back stress or shift tensor  $\alpha$  (the current center of the yield surface) which relates the Jaumann derivative of  $\alpha$  to the plastic strain rate. This incroporates effects of finite rotation and ensures objectivity of the evolution equation under rigid-body rotations. They obtained the unexpected result, for a material which strain hardens monotonically in tension, that the shear traction grows to a maximum value at a shear strain  $\gamma$  of the order unity and then oscillates with increasing strain

with a period of about six. Similar behavior was exhibited by the normal traction on the shearing planes which was initially compressive.

Of course such a variation would not occur in practice because of the onset of instability.

A study reviewed in [2] of the analytical structure of the kinematic hardening law shows that, in the case of simple shear, the use of the conventional Jaumann derivative based on the spin causes the shift tensor a to rotate continuously and this generates oscillations in the stress field. It is also shown that this analytical structure, which is currently adopted in finite-deformation elastic-plastic codes involving kinematic hardening, is not in accord with the effects of the physical micromechanisms which produce plastic flow. A modified theory consonant with these yields a monotonically increasing shear traction for the problem under discussion.

Further study commenced at Stanford University on the project now being reported revealed the approach currently accepted as valid for large deformations results in the same difficulty in the case of more general anisotropic hardening laws. The final conclusions will be reported on the continuing project at R.P.I.

At the present time the concept of kinematic hardening is the only means by which anisotropy is introduced into the plasticity laws adopted in computer programs currently in fairly wide-spread use in engineering practice. Since the importance of the Bauschinger effect in stress evaluation is well appreciated, such computer programs will undoubtably attract even more users. It is important to alert engineers to the

errors implicit in these programs when anisotropic plasticity laws are invoked in finite deformation analyses. Equally important is the fact that our findings show how those elastic-plastic programs can be readily modified to correct the error.

#### 4. COMPUTER PROGRAM DEVELOPMENT

In our earlier work a finite-element computer program named IFDEPSA (Incremental Finite Deformation Elastic-Plastic Stress Analyzer) was developed for elastic-plastic deformation and stress analysis at finite strain. Numerical accuracy and computational efficiency were emphasized throughout the code's development. IFDEPSA was first used to calculate the residual stresses generated in plane strain extrusion of a metal through frictionless curved dies. A method of handling friction at metal-tool interfaces was then developed and successfully applied.

Under this project work has continued on extending the program's capability to analyze practical metal-forming processes. Special attention has been given to the problem of modeling metal-tool interfaces for satisfactory finite-element computation, i.e. the specification and implementation of complex boundary conditions on the deforming metal's surface. One must first face the fact that the boundary conditions for a material point of the deforming metal's surface change abruptly as the point makes contact with the tool and again when separation occurs. The locations of such points are often not known a priori and must be determined as a part of the solution. In the rolling process the situation is further complicated in the region of contact by the fact that slipping occurs near the endpoints while a no-slip condition prevails elsewhere.

This means that the location of two additional points of transition must be continuously determined. Thus each surface material point encounters four boundary condition discontinuities where, for example, one traction component boundary condition abruptly becomes a condition on the corresponding velocity component. Procedures were devised for handling most of these difficulties in a finite-element context and are currently being tested on a plane strain rolling problem.

A study comparing the efficiency and accuracy of a number of finiteelements appropriate for elastic-plastic analysis was completed and the
results were presented at the Plasticity Workshop held at Stanford
University in July 1981 [5]. The so-called crossed triangle quadrilateral
element and the four noded constant dilatation quadrilateral element
were judged clearly superior to the various higher order elements considered.

Several refinements were made in IFDEPSA to further improve accuracy and efficiency. Primary among these was a reorganization of the data storage structure and the inclusion of several algorithms in the predictor-corrector loop. The capability to do axisymmetric analysis was added to the program and was merged with the plane strain capability so efficiently that almost no additional storage and very few additional lines of code were required and so that execution time for plane strain problems was virtually unaffected.

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